Örnek: $L: \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$,

$L(A) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot A$, ile tanımlansın.

$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ ve

$T = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ $\mathbb{R}^{2\times 2}$ uzayının

iki sıralı bazıdır.

a) S b) T c) S ve T d) T ve S sıralı

bazlarına göre L'nin temsilcilerini bulunuz.

a) $\mathbb{R}^{2\times 2}_S \to \mathbb{R}^{2\times 2}_S$ b) $\mathbb{R}^{2\times 2}_T \to \mathbb{R}^{2\times 2}_T$

c) $\mathbb{R}^{2\times 2}_S \to \mathbb{R}^{2\times 2}_T$ d) $\mathbb{R}^{2\times 2}_T \to \mathbb{R}^{2\times 2}_S$

a) $\mathbb{R}^{2\times 2}_S \to \mathbb{R}^{2\times 2}_S$

$$M = \left[[\underbrace{L(s_1)}]_S \;\vdots\; [\underbrace{L(s_2)}]_S \;\vdots\; \cdots \;\vdots\; [\underbrace{L(s_4)}]_S \right]$$

$$L(s_1) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = a_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + a_3 \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + a_4 \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$$

$\downarrow$

$a_1 = 1$

$a_3 = 3$

$$L(s_2) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$$

$$L(s_3) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix}$$

$$L(s_4) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \end{bmatrix} = [L]_{S,S}$$

b) $[L]_{T,T} = M_2$ (Alıştırma)

$$M_2 = \begin{bmatrix} 4 & 3 & 0 & 3 \\ -6 & -5 & -6 & -2 \\ 3 & 3 & 7 & 0 \\ 8 & 7 & 6 & 3 \end{bmatrix}$$

c) $L: \mathbb{R}^{2\times 2}_{S} \longrightarrow \mathbb{R}^{2\times 2}_{T}$

$$M_S = \left[[L(s_1)]_T \;\vdots\; [L(s_2)]_T \;\vdots\; [L(s_3)]_T \;\vdots\; [L(s_4)]_T \right]$$

$$L(s_1) = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = a_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + a_3 \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + a_4 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$L(s_2) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix} = \quad ''$$

$$L(s_3) = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} = \quad ''$$

$$L(s_4) = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} = \quad ''$$

$$\begin{array}{r|c|c|c|c} a_1 + a_2 + a_3 = & 1 & 0 & 2 & 0 \\ a_2 + a_4 = & 0 & 1 & 0 & 2 \\ a_3 = & 3 & 0 & 4 & 0 \\ a_1 = & 0 & 3 & 0 & 4 \end{array}$$

$$\left[\begin{array}{cccc|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 & 0 & 4 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 2 & 0 \end{array}\right] \overset{-S_1+S_4}{\sim} \left[\begin{array}{cccc|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 & 0 & 4 & 0 \\ 0 & 1 & 1 & 0 & 1 & -3 & 2 & -4 \end{array}\right]$$

$$\begin{array}{r} a_1+a_2+a_3 = 1 \mid 0 \mid 2 \mid 0 \\ a_2+a_4 = 0 \mid 1 \mid 0 \mid 2 \\ a_3 = 3 \mid 0 \mid 4 \mid 0 \\ a_1 = 0 \mid 3 \mid 0 \mid 4 \end{array}$$

$$-S_2+S_4 \left[\begin{array}{cccc|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 1 & -1 & 1 & -4 & 2 & -6 \end{array}\right] \overset{-S_3+S_4}{\sim} \left[\begin{array}{cccc|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & -1 & -2 & -4 & -2 & -6 \end{array}\right]$$

$$-S_4 \quad \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}\hspace{-4pt}\left|\begin{matrix} 0 \\ 0 \\ 3 \\ 2 \end{matrix}\right|\begin{matrix} 3 \\ 1 \\ 0 \\ 4 \end{matrix}\left|\begin{matrix} 0 \\ 0 \\ 4 \\ 2 \end{matrix}\right|\begin{matrix} 4 \\ 2 \\ 0 \\ 6 \end{matrix}\Bigg] \sim^{-S_4+S_2} \left[\begin{array}{cccc|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 0 & -2 & -3 & -2 & -4 \\ 0 & 0 & 1 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 2 & 4 & 2 & 6 \end{array}\right]$$

$$\downarrow$$

$$[L(S_1)]_T$$

$$\left[\begin{array}{cccc|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & -1 & -2 & -4 & -2 & -6 \end{array}\right]$$

$$M_J = \begin{bmatrix} 0 & 3 & 0 & 4 \\ -2 & -3 & -2 & -4 \\ 3 & 0 & 4 & 0 \\ 2 & 4 & 2 & 6 \end{bmatrix} : L_{S,T}$$

d) $L : \mathbb{R}^{2\times 2}_{T} \to \mathbb{R}^{2\times 2}_{S}$: $[L]_{T,S} = M_4 = ?$

(Alıştırma.)

Örnek: $L : P_1 \to P_2$, $L(p(t)) = t \cdot p(t) + p(0)$

şeklinde tanımlanıyor.

$S = \{t, 1\}$ ve $S' = \{t+1, t-1\}$ P_1 in,

$T = \{t^2, t, 1\}$ ve $T' = \{t^2+1, t-1, t+1\}$ P_2 nin

sıralı bazları ise; L nin

a) S ve T sıralı bazlarına göre,

b) S' ve T' "

temsilcilerini bulunuz.

(b) $L: \underset{(S')}{P_1} \to \underset{(T')}{P_2}$

$$M = \left[\left[L(s_1') \right]_{T'} , \left[L(s_2') \right]_{T'} \right] =$$

$$L(t+1) = t \cdot (t+1) + 1 = t^2 + t + 1 = a_1(t^2+1) + a_2(t-1) + a_3(t+1)$$

$$L(t-1) = t \cdot (t-1) - 1 = t^2 - t - 1 = \text{———— '' ————}$$

$$a_1 t^2 + (a_2+a_3)t + (a_1 - a_2 + a_3) = (t^2+t+1) \,,\, = (t^2 - t - 1)$$

$$\left.\begin{array}{l} a_1 = 1 \\ a_2 + a_3 = 1 \\ a_1 - a_2 + a_3 = 1 \end{array} \right| \begin{array}{c} 1 \\ -1 \\ -1 \end{array} \Bigg\} \left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \end{array}\right] \overset{-S_1+S_3}{\sim} \left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & -1 & 1 & 0 & -2 \end{array}\right]$$

$$\overset{S_2+S_3}{\sim}\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 2 & 1 & -3 \end{array}\right] \overset{\frac{1}{2}S_3}{\sim} \left[\begin{array}{c|c|c} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{matrix}}_{} & \begin{matrix} 1 \\ 1 \\ 1/2 \end{matrix} & \begin{matrix} 1 \\ -1 \\ -3/2 \end{matrix} \end{array}\right]$$

$$\overset{-S_3+S_2}{\sim}\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -3/2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & -1 & 1 & 0 & -2 \end{array}\right]$$

$$M=\begin{bmatrix} 1 & 1 \\ 1/2 & 1/2 \\ 1/2 & -3/2 \end{bmatrix} \checkmark$$

(a) Şıkkı

Alıştırma.

Benzerlik:

Teorem: $L : V \to W$ bir lineer dönüşüm, $boy(V) = n$ ve $boy(W) = m$ olsun. S ve S'; V nin iki sıralı bazı, T ve T'; W nun iki sıralı bazı olsun. Eğer L nin V ve W nun S ve T bazlarına göre temsilcisi A matrisi ise; $P = P_{S \leftarrow S'}$ ve $Q = Q_{T \leftarrow T'}$ geçiş matrisleri olmak üzere L nin S' ve T' bazlarına göre temsilcisi

$$B = Q^{-1} A P$$

matrisi olur.

$L: V_S \to W_T$, $[L]_{S,T} = A$

$L: V_{S'} \to W_{T'}$ $[L]_{S',T'} = B = Q^{-1} A P$

İspat: Her $x \in V$ için $\underline{[L(x)]_T = A . [x]_S}$

$[L(x)]_{T'} = Q^{-1} [L(x)]_T$

$P . [x]_{S'} = [x]_S$

$[x]_{S'} = P^{-1} [x]_S$

$(Q^{-1} = Q_{T \leftarrow T'})$

$[L(x)]_{T'} = Q^{-1} [L(x)]_T = \underbrace{(Q^{-1} A . P)}_{} [x]_{S'}$

$B . [x]_{S'} = [L(x)]_{T'}$

$\square$

$L : V \to W$

$$\begin{array}{cc} S & T \\ \hline S_1 & T_1 \\ S_2 & T_2 \\ \vdots & \vdots \end{array}$$

$$\underset{A}{\underset{\shortparallel}{[L]_{S,T}}} \overset{?}{\sim} \underset{B}{\underset{\shortparallel}{[L]_{S',T'}}}$$

$$B = Q^{-1} \cdot A \cdot P$$

$Q : T \leftarrow T'$

$P : S \leftarrow S'$

Örnek: $L : \mathbb{R}^3 \to \mathbb{R}^2$, $L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} a_1 + a_3 \\ a_2 - a_3 \end{bmatrix}$

lin. döns. veriliyor.

$\mathbb{R}^3$ ün iki sıralı bazı $S =$ Standart Baz,

$S' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ ve $\mathbb{R}^2$ nin iki

sıralı bazı $T = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ ve $T' = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$

olsun.

a) L nin S ve T sıralı bazlarına göre temsilcisi?

b) $P_{S \leftarrow S'}$ ve $Q_{T \leftarrow T'}$ geçiş matrisleri?

c) L nin S' ve T' bazlarına göre temsilcisini (a) ve (b) den yararlanarak bulunuz.

d) L nin S' ve T' bazlarına göre temsilcisini doğrudan bulunuz.

(a) $[L]_{S,T} = A = \left[\, [L(s_1)]_T \;\vdots\; [L(s_2)]_T \;\vdots\; L(s_3) \,\right]$

$$L\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} a_1+a_3 \\ a_2-a_3 \end{bmatrix}$$

$$\left.\begin{array}{l}
L\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow [L(s_1)]_T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
L\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow [L(s_2)]_T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
L\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \rightarrow [L(s_3)]_T = \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{array}\right\} A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

(b) $P_{S \leftarrow S'} = \left[[s_1']_S \,\vdots\, [s_2']_S \,\vdots\, [s_3']_S \right]$

$$a_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \,\Bigg|\, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \,\Bigg|\, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c|c|c} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(the last three columns: $P_{S \leftarrow S'}$)

$Q_{T \leftarrow T'} = Q \Leftrightarrow Q^{-1} = Q_{T' \leftarrow T}$

$\left[[t_1]_{T'} \,\vdots\, [t_2]_{T'} \right] = Q^{-1} \Rightarrow a_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{matrix} 1 \\ 0 \end{matrix} \,\Big|\, \begin{matrix} 1 \\ 0 \end{matrix} \,\Big|\, \begin{matrix} 0 \\ 1 \end{matrix}$

$$\left[\begin{array}{cc|c|c} 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$

$$-S_1+S_2 \left[\begin{array}{cc|c|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] \overset{\frac{1}{2}S_2}{\sim} \left[\begin{array}{cc|c|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 1/2 \end{array}\right]$$

$$\underset{\sim}{-S_2+S_1} \left[\begin{array}{cc|c|c} 1 & 0 & 3/2 & -1/2 \\ 0 & 1 & -1/2 & 1/2 \end{array}\right] \Rightarrow Q^{-1} = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

$$\underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{I_2} \quad \underbrace{\begin{matrix} 3/2 & -1/2 \\ -1/2 & 1/2 \end{matrix}}_{Q^{-1}}$$

$$\left[\begin{array}{cc|c|c} 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$$

$$\underbrace{\begin{matrix} 1 & 1 \\ 1 & 3 \end{matrix}}_{-\,.\,I_2}$$

c) $[L]_{S',T'} = B = Q^{-1}AP$, $\quad A = [L]_{S,T}$

$$B = \underbrace{\begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}}_{Q^{-1}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{P}$$

$$B = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 2 & 3 & 4 \\ 0 & -1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3/2 & 2 \\ 0 & -1/2 & -1 \end{bmatrix} \quad /\!/\!/ \qquad B = Q^{-1} A P$$

d) $$B = \left[[L(s_1')]_{T'} \; [L(s_2')]_{T'} \; [L(s_3')]_{T'} \right] = \left[\begin{bmatrix} 1 \\ 1 \end{bmatrix}_{T'} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{T'} \begin{bmatrix} 1 \\ -1 \end{bmatrix}_{T'} \right]$$

S', T'

$$\underbrace{a_1\begin{bmatrix}1\\1\end{bmatrix}+a_2\begin{bmatrix}1\\3\end{bmatrix}}=\begin{bmatrix}1\\1\end{bmatrix}\ \vdots\ \begin{bmatrix}1\\0\end{bmatrix}\ \vdots\ \begin{bmatrix}1\\-1\end{bmatrix}$$

$$\left[\begin{array}{cc|c|c|c}1&1&1&1&1\\1&3&1&0&-1\end{array}\right]\overset{-S_1+S_2}{\sim}\left[\begin{array}{cc|c|c|c}1&1&1&1&1\\0&2&0&-1&-2\end{array}\right]$$

$$\left[\begin{bmatrix}1\\1\end{bmatrix}_{T'}\ \vdots\ \begin{bmatrix}1\\0\end{bmatrix}_{T'}\ \vdots\ \begin{bmatrix}1\\-1\end{bmatrix}_{T'}\right]$$

$$\overset{\frac{1}{2}S_2}{\sim}\left[\begin{array}{cc|c|c|c}1&1&1&1&1\\0&1&0&-1/2&-1\end{array}\right]\overset{-S_2+S_1}{\sim}\left[\begin{array}{cc|c|c|c}1&0&1&3/2&2\\0&1&0&-1/2&-1\end{array}\right]$$

$$\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{I_2}\qquad\underbrace{\begin{matrix}1&3/2&2\\0&-1/2&-1\end{matrix}}_{B}$$

$$[L]_{S',T'}=B=\begin{bmatrix}1&3/2&2\\0&-1/2&-1\end{bmatrix}$$

Sonuç: $L: V \to V$, n boyutlu V uzayı üzerinde bir lineer operatör (dönüşüm) olsun. S ve S'; V nin iki sıralı bazı ve $P_{S \leftarrow S'} = P$ olmak üzere; L nin S bazına göre temsilcisi A ile S' bazına göre temsilcisi B arasındaki ilişki;

$$B = P^{-1} . A . P$$ dir.

$L: V \to W$
S T
S' T'

$L: V \to V$
S, S → A
S', S' → B